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TITLE   LIGHT-FRONT NUCLEAR SHELL-MODEL

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LIGHT-FRONT NUCLEAR SHELL-MODEL

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Abstract

I examine the effects of nuclear structure on high-energy, high-momentum transfer processes, specifically the EMC effect. For pedagogical reasons, a fictitious but simple two-body system consisting of two equal-mass particles interacting in a harmonic oscillator potential has been chosen. For this toy nucleus, I utilize a widely-used link between instant-form and light-front dynamics, formulating nuclear structure and deep-inelastic scattering consistently in the laboratory system. Binding effects are compared within conventional instant-form and light-front dynamical frameworks, with appreciable differences being found in the two cases.

I. Introduction

In recent years we have seen a growing awareness of the importance of light-front concepts in nuclear physics [1]. This has been stimulated in large part by the deep-inelastic scattering (DES) on nuclei made available by the EMC and SLAC collaborations. Light-front variables are natural for DES because of the large momentum transfer Q^2 of the virtual photon. But I believe that there is additional motivation for this interest, namely the perception that high-energy, high-momentum transfer processes of all types will be of increasing in importance as a source of information about nuclei. This implies that the relativistic many-body problem will play an even greater role than it has in the past for nuclear theory, and the light-front formulation of nuclear dynamics is one approach to this problem that may have special advantages.

The foundation of light-front dynamics was laid down by Dirac [2]. He showed that relativistic dynamics, formulated using the Poincare group, may be realized in any one of several ways. This includes the familiar instant form of quantum mechanics, in which the system is quantized at equal times t, but also includes the so-called light-front form, in which the system is quantized at equal light-front time $t^+ = t + x_3$ (for simplicity we will refer to "light-front time" simply as "time" in this paper), where x_i represents the Cartesian components of the position of a particle.

Since Dirac's original paper, there has been some development of light-front methods in nuclear physics within the context of the few-body problem. Seminal work by Terent'ev and his collaborators [3-6] have occurred along these lines. These methods have applied to various few-body problems [7,8] assuming that the nucleons interact through two-body potentials. Aside from this work, there has been little serious attention given to developing light-front many-body methods for nuclear problems starting from an Hamiltonian expressed in terms of the observable baryon and meson variables. This line of research is important and may provide the most appropriate connection between nuclear structure and high-energy experiments such as the recent Drell-Yan measurements [9] on nuclei.

I will report here on the formulation of a light-front shell model that I have recently worked out with Leonard Kisslinger [10]. This is part of the larger problem of developing a light-front many-body theory. The lack of such a theory is in part due to the technically awkward light-front variables in dealing with angular momentum and classifying nuclear states. The larger picture that I have in mind begins with a two-body interaction between nucleons, obtained from an appropriate meson-nucleon Hamiltonian using, perhaps, the folded-diagram methods I briefly discussed in Ref. [11]. Then, if one is then to develop nuclear structure based on many-body perturbation theory, it is necessary to adopt an unperturbed Hamiltonian H_0 describing the interaction between a nucleon of the nucleus with the average field of the other nucleons. One of the purposes of H_0 is to provide a set of basis states for evaluating corrections to energy eigenvalues and other observables in terms of the two-body interaction. As usual in many-body theory, there is some freedom in choosing the interaction U in H_0, because the same U that is added in H_0 is also subtracted from the interaction in the Hamiltonian. The object is to make the the many-body perturbation (the difference between U and the two-body interaction) as small as possible. The interaction U may be defined in a Hartree-Fock sense, or it may be a more phenomenological shell-model type interaction. One, of course, has an efficient theory if H_0 provides a good lowest-order description of the nuclear properties. Here I will not be very specific about how this many-body theory will implemented. I will concentrate rather on the starting point of the theory, namely on the choice of a possible unperturbed shell-model interaction.

The light-front variables $(p^+, p^-, \mathbf{p}_\perp)$ for a free particle given by

$$p^+ = \sqrt{\mathbf{p}^2 + m^2} + p_3 \;\;,\;\; p^- = \sqrt{\mathbf{p}^2 + m^2} - p_3 \;\;,\;\; \mathbf{p}_\perp = (p_1, p_2) \;\;. \tag{1}$$

The variables p^+ and p^- have simple transformations to $p^{+\prime}$ and $p^{-\prime}$ for boosts in the z-direction. For a collection of particles of total momentum $P^\pm = \sum_i p^\pm(i)$, this transformation may be expressed as

$$p^{+\prime}(i)/P^{+\prime} = p^+(i)/P^+ \;\;,\;\; \text{and} \;\; p^{-\prime}(i)/P^{-\prime} = p^-(i)/P^- \;\;. \tag{2}$$

One finds it convenient to introduce the variable $x_i = p^+(i)/P^+$, which is invariant under boosts. The transverse momenta $\mathbf{p}_{(i)\perp}$ also do not change by making such a boost.

In this paper, I develop the "shell model" for a simple but fictitious system of two particles of equal masses, where in this case U is the same as the two-body interaction V_{12}. I will compare results in the instant-form descriptions and its translation to the light-front in this simple system to develop an intuition for the differences between the two formulations that will arise in the many-body case. This two-body system will be referred to as a "nucleus" and the constituents of this nucleus as "nucleons," although the connection to a real system is only suggested. We find that results for the elastic form factor and especially the deep-inelastic structure function are different in the familiar instant-form descriptions and in this particular light-front description. Kisslinger and I are extending these ideas to the more realistic situation, but I will not report any of these findings here.

For two bodies of equal masses m, the Hamiltonian may be written in light-front variables as

$$H = \frac{\mathbf{P}_\perp^2}{P^+} + \frac{M_{12}^2 + mV_{12}}{P^+} \quad .$$

(3)

where V_{12} is the two-body potential, $\mathbf{P}_\perp = \sum_i \mathbf{p}_{i\perp}$, and M_{12}^2 is the square of the mass operator for two free particles,

$$M_{12}^2 = \frac{\mathbf{p}_\perp^2 + m^2}{x_1 x_2} = \frac{\mathbf{p}_\perp^2 + m^2}{\frac{1}{4}(1 - x^2)} \quad .$$

(4)

with

$$\mathbf{p}_\perp = x_1 \mathbf{p}_{2\perp} - x_2 \mathbf{p}_{1\perp} \quad , \quad \text{and} \quad x = x_1 - x_2 = 1 - 2x_2 = 2x_1 - 1 \quad ,$$

(5)

defining the relative variables. According to the prescription of Hamiltonian light-front dynamics [3], the potential V_{12} is a local function of the relative variables in Eq. (5).

In the center-of-mass system, the eigenfunctions of H are the same as the eigenfunctions of the square of the perturbed mass operator, $M_{12}^2 + mV_{12}$,

$$\left(M_{12}^2 + mV_{12}\right)\psi_n(\mathbf{p}_\perp, x) = M_n^2\psi_n(\mathbf{p}_\perp, x) \quad .$$

(6)

Furthermore, because the perturbed mass operator is a function only of the relative variables, the eigenfunctions of H separate into two factors: one for the motion of the relative coordinates, and one for the center-of-mass coordinates as follows,

$$\Psi = \psi_{c.m.}\left(\mathbf{P}_\perp, P^+\right)\psi(\mathbf{p}_\perp, x) \quad .$$

(7)

In Eq. (6), M_n coincides with the eigenvalue of H in the nuclear rest frame.

It was recognized in the early work of Terent'ev [3] that one can obtain a solution light-front dynamics by making a change of variables and making a Melosh transformation M on the spin-angle degrees of freedom,

$$\psi_n^{(L)}(\mathbf{p}_\perp, x) = \mathbf{M}\psi_n^{(I)}\left(\mathbf{p}_\perp, p_3(x, \mathbf{p}_\perp)\right)|0\rangle \quad .$$

(8)

where $|0\rangle$ is the spin of the nucleus. The relationship between the relative variables x and $\mathbf{p}_\perp$ and p_3 is

$$p_3(x, \mathbf{p}_\perp) = \frac{-x\sqrt{\mathbf{p}_\perp^2 + m^2}}{\sqrt{1 - x^2}} \quad .$$

(9)

The result of the transformations can be seen explicitly from Eqs. (4) and (6). In this case the equivalent eigenvalue equation in instant-form dynamics takes the form

$$\left(\frac{\mathbf{p}^2}{m} + V_{12}\right)\psi_n^{(I)} = \left(\frac{M_n^2}{4m} - m\right)\psi_n^{(I)} \quad .$$

(10)

and we have used the fact that the two-body potential commutes with M. This idea was implemented later for studies of the deuteron [7,8] based on the solutions of Schroedinger's equation in instant-form dynamics.

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The operators for the angular momentum are more complicated in the light-front framework than in the instant-form. However, it is a consequence of having made the Melosh rotation in Eq. (5) that one may construct the wave function in instant-form quantum mechanics. The Melosh rotation is given explicitly by $M = r_1 r_2$, where r_i is defined as

$$r_i = \frac{m + E_p + p_3 \pm i\epsilon_{j,s}\sigma_j p_{s\perp}}{[2(E_p + m)(E_p + p_3)]^{1/2}} \ .$$ (11)

$\epsilon_{j,s}$ is a short-hand notation for the antisymmetric tensor $\epsilon_{3,j,s}$, and where the sign in Eq. (11) is opposite for particles 1 and 2.

The normalization condition on the wave functions is

$$\int v_m^{a(L)}(\mathbf{p}_\perp, x) v_n^{(L)}(\mathbf{p}_\perp, x) J \frac{d\mathbf{p}_\perp dx}{(2\pi)^3} = \int v_m^{a(I)}(\mathbf{p}) v_n^{(I)}(\mathbf{p}) \frac{d\mathbf{p}^3}{(2\pi)^3} = \delta_{m,n} \ .$$ (12)

where J is the Jacobian of the transformation from the variables (p_1, p_2, p_3) to $(\mathbf{p}_\perp, x)$,

$$J = \frac{\sqrt{p_\perp^2 + m^2}}{(1 - x^2)^{3/2}} \ .$$ (13)

Note that we do not use a covariant normalization on the right-hand side of Eq. (12). Our norm is needed to assure that the orthonormality condition is the same as that for the three-dimensional harmonic oscillator in instant-form dynamics. It is sometimes convenient to think of the light-front wave function as

$$v_m(\mathbf{p}_\perp, x) = \sqrt{J} \, \psi_m^{L,C}(\mathbf{p}_\perp, x) \ ,$$ (14)

but we prefer to write the Jacobian explicitly in order to avoid problems when changing coordinate systems.

II. Harmonic Oscillator Model

We will work entirely in momentum space where the harmonic oscillator potential has the form

$$\langle \mathbf{p}'|V_{12}|\mathbf{p}\rangle = -(\ )^3 \frac{1}{2}\mu\omega^2 \nabla_p^2 \delta(\mathbf{p}' - \mathbf{p}) \ .$$ (15)

where $\mathbf{p}$ and $\mathbf{p}'$ are the relative momenta in instant-form dynamics, $\mathbf{p} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$ and $\mu = m/2$ is the reduced mass. The purpose of the two-body potential V_{12}, is to confine the nucleons in the vicinity of the nucleus. In this case the eigensolutions of Eq. (10) are expressed in terms of Hermitian polynomials with the eigenvalues give by

$$\hbar\omega \left(n_1 + n_1 + n_1 + \frac{3}{2} \right) = \frac{M_n^2}{4m} - m \ .$$ (16)

We take the normalized wave function for the ground state of our two-body system to be the lowest oscillator state, which for the instant-form description is

$$v^{(I)}(p_1, p_2, p_3) = N \exp\left[-\frac{b^2}{2}(p_1^2 + p_2^2 + p_3^2) \right] \ ,$$ (17a)

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and for the light-front description is

$$\psi^{(L)}(\mathbf{p}_\perp, x) = N \exp\left[-\frac{b^2}{2}\left(\frac{\mathbf{p}_\perp^2 + x^2 m^2}{1 - x^2}\right)\right] \quad .\tag{17b}$$

where

$$N = 2\pi\sqrt{\frac{2b^3}{\sqrt{\pi}}} \quad \text{and} \quad b^{-1} = \sqrt{\frac{-\mu}{\hbar}} \quad .\tag{18}$$

The corresponding eigenvalue is

$$\frac{3}{2}\hbar\omega = \frac{M_n^2}{4m} - m \quad .\tag{19}$$

In comparing the charge radius and deep-inelastic structure function (Sect. III), we will want to use instant-form and light-front descriptions whose wave functions have the correct asymptotic fall-off in coordinate space. The oscillator parameters should therefore be chosen to obtain agreement between the empirical binding energy E_B and theoretical binding energy. This condition gives for the instant-form description

$$E_B = \frac{3\hbar^2}{2\mu b^2} \quad ,\tag{20}$$

and for the light-front description [see Eq. (19)]

$$\frac{3\hbar^2}{2\mu b^2} = \frac{(E_B + 2m)^2}{4m} - m \quad .\tag{21}$$

Equations (20) and (21) provide the following connection between the light-front $b(L)$ and the instant form $b(I)$ values [to lowest order in $(mb)^{-2}$]

$$b(I) = b(L) + \frac{3\hbar^2}{8m^2 b(L)} \quad .\tag{22}$$

For the purpose of comparing the elastic and deep-inelastic form factors the following integral is needed, with ψ given in Eq. (17b):

$$W = \int \frac{d^2\mathbf{p}_\perp dx}{(2\pi)^3} J|\psi|^2 \left[A + Bx^2 + Cp_\perp^2 b^2 + D\frac{p_\perp^2}{m^2} + Ep_\perp^2 b^2 x^2\right]$$
$$= A + \frac{B}{2\gamma^2} + C + \frac{D}{2\gamma^2} + \frac{E}{2\gamma^2} \quad ,\tag{23}$$

where we have kept terms to lowest order in γ^{-1}, where $\gamma = mb$.

To evaluate the charge radius we calculate the elastic form factor $F_1(q^2)$,

$$F_1(\mathbf{q}_\perp^2) = \int \frac{d^2\mathbf{p}_\perp dx}{(2\pi)^3} J\psi^*\left(\mathbf{p}_\perp - \frac{1}{2}(1-x)\mathbf{q}_\perp, x\right)\psi(\mathbf{p}_\perp, x) \quad .\tag{24}$$

which is just the infinite-momentum expression of Drell and Yan [12] for the elastic form factor. $F_1(q_\perp^2)$ is an even function of $q_\perp^2$, so the charge radius is then given by

$$R_c^2 = -12 \lim_{q_\perp^2 \to 0} \frac{\partial^2}{\partial \mathbf{q}_\perp^2} F_1(\mathbf{q}_\perp^4) \quad .\tag{25}$$

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The factor of 12 results from the fact that we have used the relative variables in the expression for F_1 in Eq. (24). If our two-body system is in its ground state, so that its wave function is given by Eq. (17b). then the integral in Eq. (24) becomes

$$F_1(\mathbf{q}_\perp^2) = N^2 \int \frac{d^2\mathbf{p}_\perp dx}{(2\pi)^3} J \exp\left[-\frac{b^2}{2}\left(\frac{\mathbf{p}_\perp'^2 + x^2 m^2}{1 - x^2}\right)\right] \langle 0|M^+ M|0\rangle$$
$$\times \exp\left[-\frac{b^2}{2}\left(\frac{\mathbf{p}_\perp^2 + x^2 m^2}{1 - x^2}\right)\right] \quad , \tag{26}$$

where

$$\mathbf{p}_\perp' = \mathbf{p}_\perp - \frac{1}{2}(1 - x)\mathbf{q}_\perp \quad . \tag{27}$$

Taking first the derivative of Eq. (26) and then the limit as in Eq. (25), we find the following integral to be done

$$\frac{-b^2}{4} \int \frac{d^2\mathbf{p}_\perp dx}{(2\pi)^3} J|\dot{\psi}|^2 \left[1 + 2x^2 - \frac{1}{2}(1 + 3x^2)b^2\mathbf{p}_\perp^2 + \frac{1}{4m^2}\right] \quad , \tag{28}$$

which is easily performed by making rather straightforward changes of variables. Using Eq. (26), we find for the charge radius, to lowest order in $1/(mb)^2$,

$$R_C(L) \cong b(L)\sqrt{\frac{3}{2}}\left(1 + \frac{\hbar^2}{2(m\,b(L))^2}\right) \quad , \tag{29}$$

where $b(L)$ is given by the solution of Eq. (21). The corresponding radius $R_C(I)$ is given by

$$R_C(I) = b(I)\sqrt{\frac{3}{2}} \cong b(L)\sqrt{\frac{3}{2}}\left(1 + \frac{3}{8(m\,b(L))^2}\right) \quad , \tag{30}$$

where we have used Eq. (22). Comparing Eqs. (29) and (30) gives

$$R_C(I) - R_C(L) \cong \frac{b(L)}{8}\sqrt{\frac{3}{2}}\frac{\hbar}{(m\,b(L))^2} \quad , \tag{31}$$

showing that the RMS radius of our nucleus in the instant-form description is somewhat larger. A similar behavior may be seen in Fig. (1) of Ref. [8].

III. The EMC Effect

I now want to consider the result of DES from our two-body system. One traditionally applies convolution formulas to relate the structure function of the nucleon to the structure function of the nucleus. For a derivation of the convolution formula in light-front quantum mechanics, see, e.g., Refs. [13,14]. In instant-form quantum mechanics, there have been two approaches to the derivation of convolution formulas, a four-dimensional one, e.g., Refs. [15,16], and a three-dimensional approach [17]. The latter was obtained by a derivation parallel to the one made in Ref. [13]; the final

expression for the structure function is given by a time-ordered, linked-cluster expansion analogous to that that enters the Goldstone expansion for observables in nuclear many-body theory.

It is the purpose of this section to compare the effects of binding on the deep-inelastic structure function $F_2^A(x)$ in the instant and light-front formulations. We will utilize for this purpose the method of moments employed by Frankfurt and Strikman [18].

Let us assume that the convolution formula has the following form.

$$F_2^A(x) = \frac{1}{A} \int_0^A dz\, f_{N/A}(z)\, F_2^N(x/z) \quad . \tag{32}$$

Frankfurt and Strikman suggest expanding about $z = 1$, at which point $f_{N/A}(z)$ is expected to peak. They then show that $F_2^A(x)$ has the form

$$F_2^A(x) = F_2^N(x)I_1 + xF_2'^N(x)I_2 + \left[x F_2'^N(x) + x^2 F_2''^N(x)/2 \right] I_3 + \cdots \quad , \tag{33}$$

where the three moments I_1, I_2, and I_3 are defined as

$$I_i = \frac{1}{2} \int_0^A f_{N/A}(z)(1 - z)^{i-1} dz \quad . \tag{34}$$

In the notation of Frankfurt and Strikman, $f_{N/A}(z)/2 = z\rho_A(z)$, so the convolution formula in Eq. (32) has the correct number of factors of z. Frankfurt and Strikman point out that in some treatments of the EMC effect, the Moller flux factor (which, in the case of the deuteron, is responsible for the forward-backward asymmetry of the spectator yield ˙ the deuteron breakup) has been omitted, and that consequently the wrong normalization occurs. The omission of the Moller flux factor is equivalent to assuming a point-like target [15]. The correct normalization is

$$\frac{1}{2} \int_0^A dz\, f_{N/A}(z) = 1 \quad . \tag{35}$$

a. Light-Front Convolution Formula

The convolution formula in light-front dynamics [13] gives the simple result

$$F_2^A(x) = \frac{1}{A} \int \frac{d^3 p_1}{(2\pi)^3} \rho^{(L)}(\mathbf{p}_1) F_2^N(x/z) \qquad 0 \le x \le A \quad , \tag{36}$$

where $z = A p_1^+/P^+$. Here F_2^A is the structure function of the nucleus, F_2^N is the structure function of the nucleon (we assume that each nucleon has the same structure function), A is the number of nucleon constituents of the nucleus, and $\rho(\mathbf{p}_1)$ is the density of one of the nucleons.

$$\rho^{(L)}(\mathbf{p}_1) = 2 \int \frac{d^3 p_2}{(2\pi)^3} \left| \Psi^{(L)}(\mathbf{p}_1, \mathbf{p}_2) \right|^2 \quad . \tag{37}$$

It is understood that delta functions conserving the center-of-mass momentum occur as needed. The factor of 2 arises because there are two nucleons in the nucleus. Clearly, $\rho(\mathbf{p}_1)$ is normalized so that

$$\int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \rho^{(L)}(\mathbf{p}_1) = 2 \quad . \tag{38}$$

The Melosh rotation has been ignored, which is proper for unpolarized scattering measurements from a spin-zero target. We will assume that our two-nucleon system is spinless.

We introduce the distribution function for nucleons, $f_{N/A}(z)$ as

$$f_{N/A}(z) \equiv \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \rho^{(L)}(\mathbf{p}_1)\delta\left(z - \frac{A\,p_1^+}{P^+}\right) \quad , \tag{39}$$

which is normalized according to Eq. (35). Using Eq. (37), this becomes

$$f_{N/A}(z) = 2 \int \frac{d^2\mathbf{p}_\perp dx}{(2\pi)^3} J \left|\psi^{(L)}(\mathbf{p}_\perp, x)\right|^2 \delta(z - x - 1) \quad , \tag{40}$$

where we have used Eq. (5) to set

$$\frac{A\,p_1^+}{P^+} = 1 + x \quad . \tag{41}$$

Using Eq. (34) we have

$$I_i = \int \frac{d^2\mathbf{p}_\perp dx}{(2\pi)^3} J \left|\psi^{(L)}(\mathbf{p}_\perp, x)\right|^2 x^{i-1} \quad , \tag{42}$$

and from Eq. (23), we find

$$I_1 = 1 \quad , \quad I_2 = 0 \quad , \quad I_3 = \frac{1}{2\gamma^2} \quad . \tag{43}$$

b. Instant-Form Convolution Formula, Four-Dimensional

The form which has been used to obtain binding effects using instant-form wave functions [15,16] is

$$F_2^A(x) = \frac{1}{A} \int_0^A dz\, F_2^N(x/z) f_{N/A}(z) \quad , \tag{44}$$

where

$$f_{N/A}(z) = 2 \int \frac{d^4p}{(2\pi)^4} \delta\left(z - \frac{A\,p^+}{P^+}\right) \frac{p^+}{\sqrt{\mathbf{p}^2 + m^2}} \left|\psi^{(I)}(p)\right|^2 2\pi\,\delta(p_0 - m + \bar{\epsilon}) \quad , \tag{45}$$

and $\bar{\epsilon} = \epsilon_D + \langle p^2 \rangle/(2m)$, with ϵ_D the binding energy of the two-body system and $\langle p^2 \rangle$ the average relative momentum of the two nucleons. Clearly, $f_{N/A}(z)$ does not exactly satisfy the normalization of Eq. (35) if integrated over the interval from 0 to 2, but as we shall see below, there is very little mistake. Performing the integrations over the delta functions gives

$$f_{N/A}(z) = M_D \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^3} \left|\psi^{(I)}(\sqrt{p_\perp^2 + (M_D/2)^2(z - \eta)^2})\right|^2 \quad . \tag{46}$$

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where $\eta = 2(m - \tilde{\epsilon})/M_D$, where we have dropped the relativistic correction $p^+/\sqrt{\mathbf{p}^2 + m^2}$ following Miller (Ref. [16]) and where we have used our normalization conventions. In this form, the result is practically the same as that of Frankfurt and Strikman [18].

For a Gaussian wave function the integral in Eq. (46) may be performed, giving

$$f_{N/A}(z) = \frac{M_D b}{\sqrt{\pi}} e^{-(M_D/2)^2 b^2 (z - \eta)^2} \quad . \tag{47}$$

Using Eq. (34) and Eq. (23) and taking $1 - \eta \equiv \delta = (\epsilon_D/M_D) + (\langle p^2\rangle/m\, M_D)$, we find

$$I_i = \frac{M_D b}{2\sqrt{\pi}} \int_{-1}^{1} dz\, z^{i-1}\, e^{-(M_D/2)^2 b^2 (z - \delta)^2} \quad . \tag{48}$$

Evaluating Eq. (48) gives

$$I_1 = \exp[-(M_D b \delta/2)^2] - \frac{2}{M_D b \sqrt{\pi}} \exp\left[-(M_D b \delta/2)^2 + \left(\frac{M_D b}{2}\right)^2 \right] \quad , \tag{49}$$

which is unity as long as $M_D b \delta \cong 0$ and $M_D b \gg 1$. These conditions are satisfied for the physical deuteron, and we assume the same to be true for our system. Under this condition, and assuming that $M_D = 2m$, we obtain

$$I_2 = \delta = \frac{\epsilon_D}{2m} + \frac{\langle p^2 \rangle}{2m^2} \quad \text{and} \quad I_3 = \frac{1}{2\gamma^2} + (1 - \eta)^2 \quad . \tag{50}$$

Note that I_3 is the same in the instant and light-front forms, but unlike the light-front result, I_2 is nonzero.

c. Instant-Form Convolution Formula, Three-Dimensional

An off-shell extension of an amplitude is usually found to be necessary when one embeds the amplitude in a medium. This situation occurs for the deep-inelastic structure function. The derivations of convolution formulas using four-dimensional approaches have the drawback that knowledge of the dependence of the nucleon structure function on p_0, the fourth component of the momentum of the struck constituent, is not known. The off-shell extrapolation of the structure function must therefore be dropped in these approaches, as stressed by Jaffe in Ref. [14]. In Ref. [17] it was noted that the off-shell extension could be accomplished without making any arbitrary assumptions if one works within a three-dimensional formulation.

The instant-form convolution formula thus derived in [17] is given by

$$F_2^A(x) = \frac{1}{A} \frac{z'}{z} \int \frac{d^3 \mathbf{p}_1}{(2\pi)^3} \rho(\mathbf{p}_1) F_2^N(z'/z) \quad , \tag{51}$$

where z' is a shifted value of z defined as

$$z' = z + \left\langle \frac{2\sqrt{p^2 + m^2} - M_D}{2m} \right\rangle \quad , \tag{52}$$

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and $\rho(\mathbf{p}_1)$ is

$$\rho^{(I)}(\mathbf{p}_1) = 2 \int \frac{d^3 \mathbf{p}_2}{(2\pi)^3} \left| \Psi^{(I)}(\mathbf{p}_1, \mathbf{p}_2) \right|^2 \quad, \tag{53}$$

which has the same normalization as Eq. (35). One also introduces the quantity $f_{N/2}(z)$ related to $\rho^{(I)}(\mathbf{p}_1)$ as in Eq. (33), giving

$$F_2^A(x) = \frac{1}{2} \frac{x'}{x} \int dz \, f_{N/2}(z) \, F_2^N(x'/z) \quad. \tag{54}$$

where

$$f_{N/A}(z) = 2 \int \frac{d^2 p_\perp}{(2\pi)^2} \, J \left| \psi^{(I)}(\mathbf{p}_\perp, z-1) \right|^2 \quad. \tag{55}$$

with $p_3(x, \mathbf{p}_\perp)$ given as in Eq. (9).

The value of x in F_2^A is shifted because the excitation energy in the residual nucleus reduces the energy available to excite the nucleon by a small amount. This reduction in energy takes the structure function off-shell, and it was shown to be possible to account for this in Ref. [17] by shifting the energy of the photon in the expression for $F_A^{\mu\nu}$. The information needed to make the off-shell extrapolation of the structure function of the nuclear constituents can therefore be determined from experiments performed on a free constituent, i.e., the separate dependence of F_2^N on the energy and three momentum of the photon.

The dependence of $F_2^A(x)$ on the moments given in Eq. (34) in this case follows very closely the light-front result of Sect. III.a; that is, it is given by Eq. (33), except that we must make the replacement

$$F_2^{N(i)}(x) \rightarrow \frac{x'}{x} F_2^{N(i)}(x') \quad, \tag{56}$$

where $F_2^{N(i)}(x')$ means the i$^{\text{th}}$ derivative with respect to x'. The integrals I_i therefore have the same value as given in Eqs. (43). [Although in principle one must use $b(I)$, Eq. (20), there is no difference between using $b(I)$ and $b(L)$ to lowest order in $1/\gamma^2$.]

It has already been noted that the shifted value of x in Eq. (51) arises from the off-shell extension of the structure function of the constituent. It is perhaps worth mentioning that gauge invariance is preserved because the energy-shift is applied uniformly for all photon momenta in $F^{\mu\nu}$, including those that occur in the tensors that relate $F^{\mu\nu}$ to F_1 and F_2, where $F^{\mu\nu}$ is the structure function for the nuclear constituent. Thus, there is a close connection between gauge invariance and the overall factor of x'/x that occurs when F_2^A is projected out of $F_A^{\mu\nu}$ using the projection operators in Eq. (2b) of Ref. [17]. This same projection procedure also leads to a factor of z (the Moller flux factor discussed in Ref. [18]), which cancels against the factor of $1/z$ in Eq. (3) of Ref. [17], thereby giving rise to the correct normalization, Eq. (35). The appearance of this $1/z$ was criticized (and does not appear) in the otherwise quite similar instant-form convolution formula of Ref. [19].

IV. Summary and Conclusions

I have discussed the light-front shell model and compared results for a toy model consisting of two equal mass particles interacting through a harmonic-oscillator potential [10] in instant-form and light-front dynamics. The oscillator parameter was fixed by the requirement that the binding energy of the ground state be the same in both cases. I have been particularly interested in the electromagnetic form factor and the deep-inelastic structure function. The RMS charge radii were sho n to differ in the instant-form and light-front descriptions by small relativistic corrections, on the order of $1/\gamma^2$, where $\gamma = mb$ with m the mass of one of the constituents and b the value of the oscillator parameter. (Using values corresponding to the physical deuteron, $\gamma \simeq 10$, and the differences of the charge radii are on the order of a few percent.) For the deep-inelastic structure function, however, much more substantial differences were found.

The relationship between DES and nuclear structure is expressed in our work by conventional convolution formulas. The ructure function is then characterized in terms of a moment expansion suggested by Frankfur: and Strikman [18]. The differences in the instant-form and light-front formulations showed up in a comparison of the first three moments I_i. Corrections linear in the binding energy appeared in instant-form and were found to be much larger than the relativistic corrections that characterize the differences in charge radii. We regard these differences as a serious matter, because they represent conflicting assessments of the role of binding effects in explaining DES data. This in turn influences the conclusion about the role of non-nucleonic contributions in nuclei.

What is one to make of these different results? I personally believe that they reflect the omission of various higher-order corrections to the theory, which would naturally arise in a systematic many-body description in both the light-front and the instant-form description. The convolution formulas from which we obtained our results are based on the impulse approximation, whose criteria of validity are different in the light-front and instant-form descriptions, as stressed in Ref. [20]. A certain class of the many-body corrections were discussed in Ref. [17] in instant-form dynamics. In addition to these, there would be corrections analogous to familiar exchange currents in nuclear physics. Undoubtedly corrections would also arise in a complete many-body theory in light-front dynamics, although the classification of these would surely be different, in part due to well-known differences in the role of vacuum excitations. The crucial tasks for the future are to to obtain a deeper understanding of these differences and thereby develop a well-founded connection between nuclear structure and DES observables.

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